A Class of Airfoils Having Finite Trailing-Edge Pressure Gradients

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Some new developments relevant to the design of single-element airfoils using potential flow methods are presented. In particular, the ramifications of the unbounded trailing-edge pressure gradients generally present in the potential flow solution for the flow over an airfoil are examined, and the conditions necessary to obtain a class of airfoils having finite trailing-edge pressure gradients are developed. The incorporation of these conditions into the inverse method of Eppler for the design of low-speed airfoils is discussed, and a design generated using the modified scheme is presented for consideration. A detailed viscous analysis of this airfoil demonstrates a significant reduction in the strong viscous-inviscid interactions generally present near the trailing edge. These reductions offer the possibility of improved airfoil performance, as well as the possibility of improved accuracy in the methods of airfoil design and analysis.

Nomenclature

c = chord length
C_d = drag coefficient
C_ℓ = lift coefficient
C_m = moment coefficient about the quarter-chord point
F = complex potential function
i = $\sqrt{-1}$
p = pressure
R = complex pressure gradient
Re = Reynolds number based on chord
U = freestream velocity
u,v = x and y components of velocity, airfoil plane
V = local velocity
w = complex velocity
z = complex variable, airfoil plane
α = angle of attack (relative to zero-lift line unless otherwise noted)
Γ = circulation
ζ = complex variable, circle plane
ρ = density

Subscripts

T = trailing-edge value
x,y = partial derivatives with respect to x and y components, respectively

Superscripts

$(\ )'$ = derivative with respect to ζ
$\overline{(\ )}$ = complex conjugate

Received Dec. 11, 1984; presented as Paper 85-0206 at the AIAA 23rd Aerospace Sciences Meeting, Reno, NV, Jan. 14-17, 1985; revision received Sept. 19, 1985.

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Introduction

MUCH of the current research effort applied to low-speed airfoils is directed toward the analysis and design of multielement sections that incorporate high-lift devices such as multiple-slotted flaps and movable leading-edge slats. The use of such airfoils permits a broad range of performance through the integration of an airfoil suitable for high-speed cruise with a configuration capable of high-lift for takeoff and landing. In spite of the strong interest in multielement designs, considerable motivation remains for the study of single-element wing sections. For example, a number of applications exist, including low-speed recreational aircraft, sailplanes, propeller blades, helicopter and windmill rotors, and aircraft in the expanding arena of remotely piloted vehicles (RPVs) for which a high-lift system is simply unnecessary or cannot be justified in view of the cost and complexity. Furthermore, much of the increased understanding resulting from the study of single-element airfoils is directly applicable to the individual components of multielement designs.

Due to the still-existing difficulties in solving the fully viscous equations, potential flow methods, coupled with boundary-layer analysis techniques, remain important tools for the design of low-speed airfoils. The most common methodology for designing such airfoils involves relating the aerodynamic performance features sought to characteristics of the boundary layer and, in turn, specifying the velocity distribution around the airfoil necessary to achieve those particular characteristics. Once the required velocity distribution has been established, the airfoil is obtained by any one of a number of inverse (design) procedures such as the potential flow method based on complex functions of Eppler.[1] At this point, the real fluid behavior of the section is ascertained using a viscous analysis method and, if the performance is not as expected, the specified potential flow velocity distribution is modified and the procedure repeated until the desired levels of performance are achieved.

Because the high levels of performance promised by a number of recent airfoil designs push the fluid behavior to very critical limits, it is important that the velocity distribution obtained in the real flow is not appreciably altered from that specified in the design process. Some of the principal difficulties in achieving reliable predictions are due to the strong viscous-inviscid interactions that occur in the vicinity of the trailing edge of an airfoil. In addition to producing major alterations to the velocity distributions calculated using potential flow methods, these interactions cause the conventional boundary-layer correction, based on displacement surface concepts, to become invalid in the region of the trailing edge. This breakdown can be attributed to pressure gradient singularities that occur at the trailing edge of the inviscid flow solution. As conventional boundary-layer theory makes use of the assumption that normal pressure gradients through the boundary layer and wake are negligible, it is clear that this assumption does not hold in the vicinity of the trailing edge, where the pressure gradients predicted from the outer solution approach infinity.

One approach to account for the strong viscous influences in the trailing-edge region is that taken by Melnik and his coworkers,[2] in which the boundary-layer concept is modified to include the effects of normal pressure gradients, wake curvature, and wake thickness. This effort has resulted in the viscous analysis code for transonic airfoils called GRUMFOIL.[3,4] With regard to the design of low-speed airfoils, the somewhat different approach to these problems described in this paper is the development of a class of airfoils in which the trailing-edge singularities appearing in the inviscid flow solution are removed. For this new class of airfoils, it is hoped that the reduction of the strong viscous-inviscid interactions will result not only in closer agreement between the potential flow and the real flow velocity distributions but also in conventional boundary-layer theory being sufficient to predict accurately the viscous flow behavior. More importantly, by forcing the inviscid trailing-edge pressure gradients to be bounded, it is hoped that in the real flow the fluid will be able to pass from the airfoil into the wake more smoothly than is otherwise the case. In so doing, not only are the critical pressure recoveries of modern designs more likely to be realized without suffering unpredicted flow separations, but the possibility is offered for some significant gains in airfoil performance.

Flow in the Vicinity of the Trailing Edge

In addition to the presence of infinite trailing-edge pressure gradients, another aspect of flow in the vicinity of the trailing edge that warrants consideration is the use in airfoil design of potential flow velocity distributions characterized by a large velocity differential between the upper and lower surfaces over the aft portions of the airfoil in the region of the trailing edge. As demonstrated by the Kennedy and Marsden[5] airfoil shown in Fig. 1, this differential is introduced as it increases the area enclosed by the velocity distribution, which results in increased lift production. Note that although a nearly vertical line closes the velocity distribution at the trailing edge, a truly vertical closure is prohibited by the fact that potential flow theory requires that each point in the flowfield have a unique velocity. Consequently, in the potential flow velocity distribution there can be no velocity difference at the point where the flows from the upper and lower surfaces meet and flow into the wake. Although the upper surface pressure recovery distributions of a number of airfoils appearing in the literature, including that of Fig. 1, have been formulated using a value of V_T/U in excess of unity, this is not, in actuality, the value of the trailing edge; rather, it corresponds to that of a location on the upper surface slightly upstream of the trailing edge. From this point, the fluid is decelerated very rapidly to the actual velocity at the trailing edge. This value must be equal to that resulting from accelerating the flow on the lower surface in the immediate vicinity of the trailing edge through a steep, favorable pressure gradient. While it is quite clear that the viscous effects prevent the full realization of the lift gains predicted by potential flow methods, experimental results have indicated that for some applications the proper implementation of large velocity differentials between the upper and lower surfaces to very near the trailing edge, although usually accompanied by some drag penalty, can be of some lift benefit.[5]

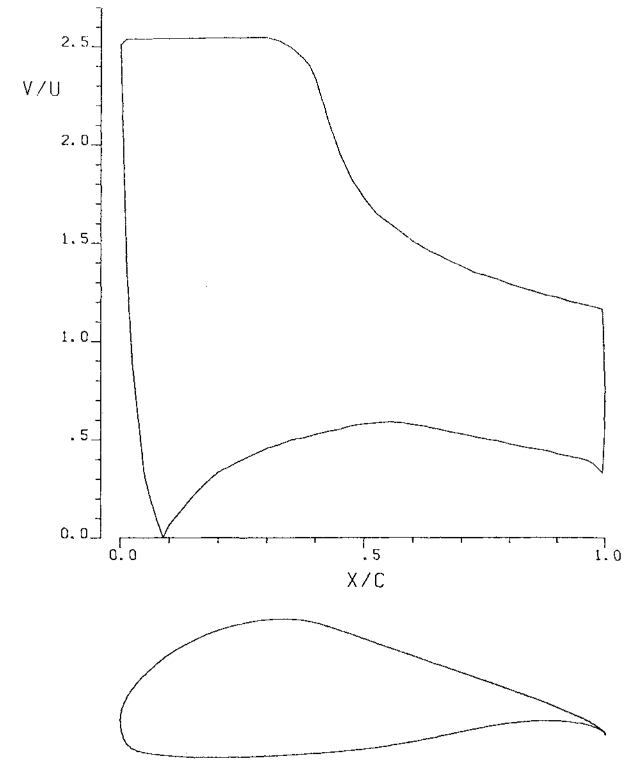

Fig. 1 Thirty-one percent thick Kennedy and Marsden high-lift airfoil and design velocity distribution; $Re = 1 \times 10^6$.

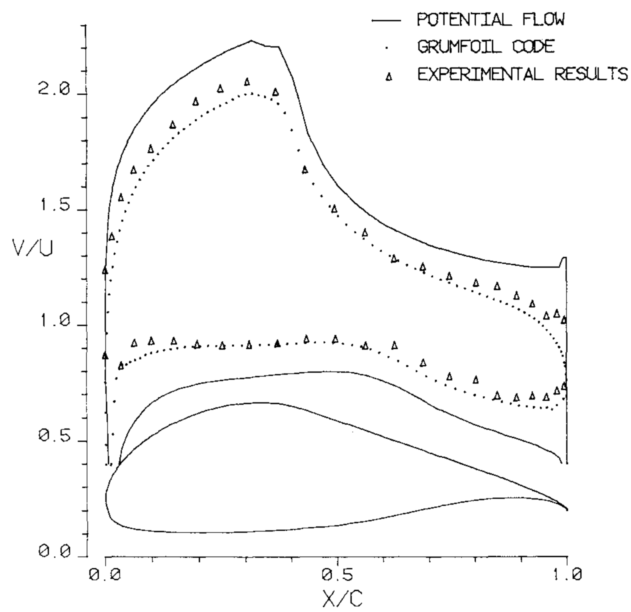

Fig. 2 Kennedy and Marsden airfoil off-design potential flow velocity distribution (Eppler panel method code) compared with viscous analysis (GRUMFOIL code) and experimental results; $\alpha = 4.2$ deg (relative to chord line), $Re = 1 \times 10^6$.

To further demonstrate the real fluid influences, Fig. 2 presents an off-design potential flow velocity distribution of the Kennedy and Marsden airfoil along with one obtained experimentally.[5] Also included in Fig. 2 is the fully viscous velocity distribution obtained using the GRUMFOIL code. It is clear in this case that the potential flow velocity distribution over the airfoil is modified considerably by viscous effects. In particular, the steep trailing-edge velocity gradient is rounded off to the extent that the actual trailing-edge velocity is easily identified. Also, note that the overall upper surface recovery has become somewhat steeper and can dramatically influence the airfoil performance. In order to maximize lift, the airfoil under consideration was designed to have a Stratford recovery distribution which, theoretically, recovers a given amount of pressure in the shortest distance.[6] The path that achieves this end is one in which separation is imminent along its entire length and, in this regard, is the steepest unseparated recovery possible. Consequently, when operating at the design condition, the steepening of the recovery gradient due to viscous effects is likely to result in severe separation problems. Even in the case of airfoils not pushing the recovery limits of a Stratford distribution, only the gentlest recoveries will have sufficient momentum in the boundary layer to overcome the steepened adverse pressure gradient that is generally introduced in the vicinity of the trailing edge by the presence of large amounts of aft loading. In any event, this example demonstrates that because viscous influences can so dramatically alter the performance from that predicted using potential flow velocity distributions, the design of high-performance airfoils must, in some way, integrate viscous flow considerations into the potential flow design method.

In further considering the modification of the potential flow velocity distributions for maximum lift by viscous effects, there clearly is a trade-off between the lift gained by maintaining large amounts of aft loading and the lift lost by separation. While there are examples of highly aft-loaded potential flow designed airfoils that achieve some portion of their design goals,[5,7] others exhibit extremely poor performance, attributed primarily to widespread flow separation.[8,9]

In the context of designing airfoils having predictable characteristics, the inconsistency of results for airfoils having large velocity differentials between the upper and lower surfaces near the trailing edge leaves much to be desired. As noted in Ref. 10, some of this uncertainty can be eliminated by reducing the amount of upper surface aft loading. Furthermore, it is found that the reduction of the viscous interactions in the trailing-edge region brought about by the introduction of the condition to ensure bounded trailing-edge pressure gradients is of value. Not only does the enforcement of this condition permit the fluid on the airfoil to flow smoothly into the wake but, in addition, because of its global influence in controlling the manner in which the flow approaches the trailing edge, it typically excludes the possibility of the closure contribution causing steep upper surface adverse gradients in the vicinity of the trailing edge. Consequently, the elimination of steep aft adverse pressure gradients should not only be beneficial to airfoil performance but to improved performance prediction as well.

Conditions Required for Finite Trailing-Edge Pressure Gradients

To examine the flow in the vicinity of the trailing edge in detail, consider, as depicted in Fig. 3, the transformation of a unit circle centered at the origin of the ζ plane into an airfoil in the z plane. The complex potential function for the unit circle having circulation Γ and an angle of attack α is

$$F(\zeta) = Ue^{-i\alpha}\zeta + Ue^{i\alpha}\zeta^{-1} - (\Gamma/2\pi i)\ln\zeta \tag{1}$$

and the complex velocity is

$$w(\zeta) = \frac{dF(\zeta)}{d\zeta} = Ue^{-i\alpha} - Ue^{i\alpha}\zeta^{-2} + \frac{i\Gamma}{2\pi}\zeta^{-1} \tag{2}$$

In order to satisfy the Kutta condition requiring smooth flow off the trailing edge, it is necessary to fix the rear stagnation point in the circle plane at $\zeta = \zeta_T = 1$. The circulation necessary to do this is found to be

$$\Gamma = 4\pi U \sin\alpha \tag{3}$$

Using this result, the complex velocity may be written as

$$w(\zeta)/U = (e^{-i\alpha} + e^{i\alpha}\zeta^{-1})(1 - \zeta^{-1}) \tag{4}$$

and, making use of the transformation function $z = z(\zeta)$, the velocity in the airfoil plane given by

$$w(z) = u - iv = \frac{dF}{d\zeta}\,\frac{d\zeta}{dz} = w(\zeta)\ z'(\zeta) \tag{5}$$

As this study is concerned with nonzero trailing-edge velocities, it is necessary that the trailing edge of the airfoil be cusped. This requires that

$$z_T{}'(\zeta) = 0 \tag{6}$$

Using Eq. (5), it is found that the expression for the velocity at the trailing edge is of an indeterminate form. The use of l'Hôpital's rule yields

$$w(z_T) = w'(\zeta_T)/z_T{}'' \tag{7}$$

At this point, the complex pressure gradient is defined as

$$R = \frac{\partial p}{\partial x} - i\frac{\partial p}{\partial y} = p_x - ip_y \tag{8}$$

which, by making use of the Bernoulli equation, may be expressed

$$R = -\rho\bar{w}(z)w'(z) \tag{9}$$

At the trailing edge, the complex pressure gradient is given by

$$R_T = -\rho\bar{w}(z_T)w'(z_T) \tag{10}$$

In order to evaluate Eq. (10), an expression for $w'(z_T)$ is required. Eq. (7) and applying l'Hôpital's rule twice to the resulting indeterminate forms gives

$$w'(z_T) = \lim_{\zeta\to\zeta_T}\left[\frac{w''(\zeta)z'' - w'(\zeta)z'''}{6(z')(z'')^2 + 3(z')^2z'''}\right] \tag{11}$$

Because the denominator of this expression is zero, $w'(z_T)$ will be unbounded, as is generally the case, unless the numerator is also zero. Thus, for $w'(z_T)$ to be finite, it is required that

$$z'''{}_T/z''{}_T = w''(\zeta_T)/w'(\zeta_T) \tag{12}$$

If this condition is satisfied, then Eq. (11) will be of an indeterminant form and, using l'Hôpital's rule once again, yields

$$w'(z_T) = [w'''(\zeta_T)z_T{}'' - w'(\zeta_T)z_T^{iv}]/3(z_T'')^3 \tag{13}$$

Substituting the above relation along with the conjugate of Eq. (7) into Eq. (10), the value of the complex trailing-edge pressure gradient, provided that Eq. (12) is satisfied, is given by

$$R_T = \frac{-\rho\bar{w}(z_T)}{3\bar{z}''{}_T(z_T'')^3}[z_T{}''w'''(\zeta_T) - z_T^{iv}w'(\zeta_T)] \tag{14}$$

To further evaluate Eq. (12), Eq. (4) may be differentiated successively and the results used to express the condition required for the complex trailing-edge pressure gradient to be finite as

$$z_T'''/z_T'' = -3 - i\tan\alpha \tag{15}$$

While the ramifications of this requirement will be considered later, note that its imaginary part can only be satisfied for a given airfoil at a single angle of attack. At any other angle of attack, the complex pressure gradient is unbounded.

In a manner similar to that used in obtaining the above expression, the value of the complex trailing-edge pressure gradient, from Eq. (14), is found to be

$$R_T = \frac{-4\rho U^2 \cos\alpha}{3\bar{z}_T''(z_T'')^3}\left[6 z_T''(2\cos\alpha + i\sin\alpha) - z_T^{iv}\cos\alpha\right] \tag{16}$$

provided that the transformation function meets the condition given by Eq. (15).

Design Method

In order to generate airfoils making use of finite trailing-edge pressure gradients, it is necessary to incorporate the required condition into some airfoil design methodology. To this end, the inverse scheme developed by Eppler,[1] and used in the airfoil design and analysis code of the Eppler and Somers,[11] was selected as being the most suitable for this purpose. An attractive capability of this method is that the formulation allows for different parts of the airfoil to be specifically designed for different angles of attack. Because of this, it is possible to use an adapted version of the Eppler Somers code in such a manner that the condition for finite trailing-edge pressure gradients can be met with minimum impact to the airfoil geometry resulting from other design requirements. The details of the development of the modified version of the code and its usage are presented in Ref. 12.

It should be emphasized that the airfoil that results from the transformation function $z = z(\zeta)$ is a solution to an elliptic problem. Thus, the condition imposed by Eq. (15) has a global influence on the transformation function and the resulting profile. For an airfoil whose mapping function is not close to satisfying Eq. (15), imposing the condition for finite trailing-edge pressure gradients can have a dramatic impact on the geometry of the profile. In most cases, however, it has been found that an airfoil can be adapted to have finite trailing-edge pressure gradients with only minor modifications.

Examples and Applications

To explore the ramifications of satisfying the condition prescribed for finite trailing-edge pressure gradients, a number of airfoils that have appeared in the literature were redesigned using the modified version of the Eppler and Somers code. An example of an airfoil considered in this manner is the well-proven and documented Wortmann FX 67-K-150, presented in Fig. 4, as defined by the aerodynamically smoothed coordinates given in Ref. 13. Although this section is intended for use with flaps, only the neutral flap configuration is considered here. For airfoils not having steep velocity gradients near the trailing edge, the conditions required to achieve finite trailing-edge pressure gradients are achievable with only minor modifications to the parent airfoil. In the case of the Wortmann section, however, due to the differential between velocities on the upper and lower surfaces near the trailing edge, a steep adverse pressure gradient is present over the aft portion of the velocity distribution. Consequently, in redesigning this airfoil, the goal was to have finite trailing-edge pressure gradients as well as to eliminate the upper surface aft loading. Thus, the airfoil obtained, although having a somewhat altered velocity distribution, should embody the same design philosophy as the original airfoil and achieve comparable performance.

With the preceding thoughts in mind, consider the result shown in Fig. 5 and note that the use of the modified code has changed the velocity distribution such that the upper surface aft loading present on the Wortmann section, causing the steep adverse pressure gradient near the trailing edge, has been replaced by a steep favorable gradient in that region on the lower surface. While it was found that the drag could be reduced considerably by beginning the lower surface

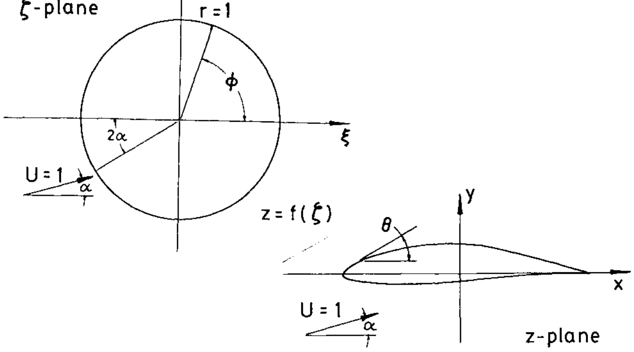

Fig. 3 Transformation of unit circle into an airfoil.

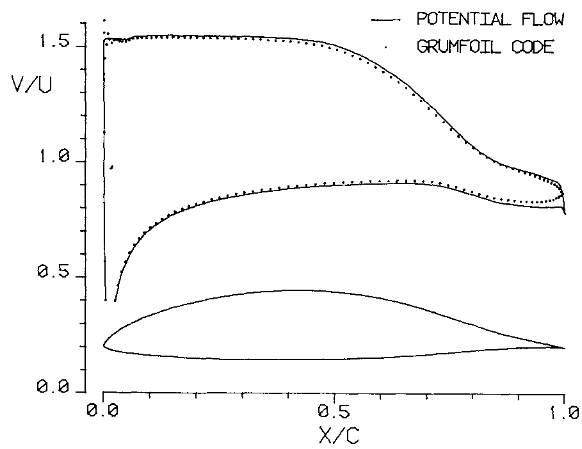

Fig. 4 Aerodynamically smoothed Wortmann FX 67-K-150 airfoil and calculated velocity distributions; $\alpha = 10.0$ deg.

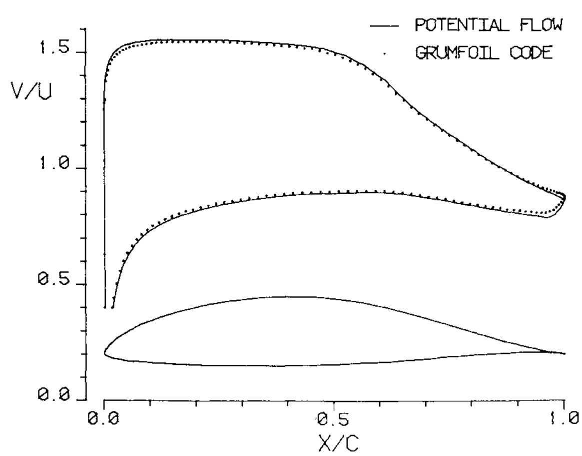

Fig. 5 Finite trailing-edge pressure gradients airfoil based on Wortmann FX 67-K-150 and calculated velocity distributions; $\alpha = 10.0$ deg.

recovery sooner, and thereby lessening the gradient, the distribution shown was retained as it is more like that of the original Wortmann section. In addition, in order to control separation problems introduced by the velocity distribution changes at the trailing edge, some modifications to the upper surface recovery distribution were necessary. Although the elimination of the upper surface aft loading results in a loss of the potential flow calculated lift, this is offset by the increased loading present on the lower surface.

A comparison of the overall aerodynamic performance of the FX 67-K-150 and the airfoil based on it is provided by the viscous analysis results obtained using the Eppler and Somers code, presented in Figs. 6 and 7. Both airfoils exhibit best lift-to-drag ratios at a lift coefficient near unity. A more detailed comparison reveals that the drag polars of the new profile are roughly equivalent to those of the original section over most of the usable performance range, although the performance of the section generated with the modified code is extended somewhat in the direction of higher lift coefficients.

While the imaginary part of the condition necessary for an airfoil to have finite trailing-edge pressure gradients can only be satisfied at a single angle of attack, as Eq. (15) reveals, the results of the Eppler and Somers viscous analysis indicate nothing particularly special about the aerodynamic characteristics at that angle of attack. This fact, however, should be expected in that the analysis makes use of conventional boundary-layer theory in which normal pressure gradients through the boundary layer, as well as wake influences, are assumed to be negligible. It should be no surprise then that the results of a calculation based on conventional boundary-layer theory do not indicate any characteristics attributable to the presence of finite trailing-edge pressure gradients.

A thorough evaluation of the effect of finite trailing-edge pressure gradients on airfoil performance requires a fairly extensive investigation, which makes use of a theoretical model having a detailed description of the flow in the vicinity of the trailing edge. To demonstrate what such a model might indicate, the GRUMFOIL code was utilized to analyze the two airfoils just presented. The Reynolds number used for these calculations was 2×10^6 and the Mach number was set to zero. The resulting velocity distributions, modified by viscous effects, are included for comparison with the Eppler and Somers potential flow results in Figs. 4 and 5. In considering the influence of viscous effects on the velocity distribution of the Wortmann airfoil, it should be observed in Fig. 4 that the potential flow prediction is modified considerably in the vicinity of the trailing edge by the action of viscosity. As seen in Fig. 5 for the airfoil having finite trailing-edge pressure gradients, on the other hand, the inviscid calculations are much less impacted by the inclusion of viscous-inviscid interactions.

The aerodynamic performance of the Wortmann airfoil as predicted by the GRUMFOIL code is summarized by the indicated points on the plot in Fig. 6 and those of the modified section in Fig. 7. Experience has shown that the transition point predictions of the Eppler and Somers code are, in general, somewhat more conservative than those obtained using GRUMFOIL. Thus, to facilitate the comparison of the performance predictions obtained from the two codes, the lower surface transition points were fixed for the GRUMFOIL calculations at the locations predicted by the Eppler and Somers code. Even so, the drag predicted using GRUMFOIL is consistently less over the entire operating range than that of the Eppler and Somers analysis. The divergence of the two calculations at higher lift coefficients is due to the fact that the Eppler and Somers code calculations account for separation to some degree, while the GRUMFOIL results shown do not. As with the Eppler and Somers results, the GRUMFOIL predictions also indicate that the modified section has a slightly better lift-to-drag ratio than does the original design. It must again be emphasized, however, that no attempt was made to maximize the performance of the modified section relative to that of the parent airfoil but, rather, the effort was directed at matching the velocity distributions as closely as possible.

Because the imaginary part of the condition required for achieving finite trailing-edge pressure gradients can only be satisfied at a single angle of attack, as has been noted, it is of interest to note that, based on the off-design performance presented in Fig. 7, this limitation does not appear to be significant. It is apparent from these results, as well as a detailed study of a number of other off-design velocity distributions and boundary-layer calculations,[12] that strong viscous interactions at the trailing edge do not dramatically appear when a finite trailing-edge pressure gradients airfoil is operated off-design. Thus, on the basis of these studies, it is concluded that if aerodynamic benefits are realized by the presence of finite trailing-edge pressure gradients, then these benefits are not limited to the design angle of attack but are present over an operational range of angles.

Further differences in the flow behavior in the trailing-edge region between the two airfoils are well demonstrated by comparing the development of the boundary-layer displacement thicknesses as calculated using the GRUMFOIL code and presented in Fig. 8. In the case of the Wortmann airfoil, the singular behavior at the trailing edge is readily observed. It should be noted that these characteristics conform very well to those typically found experimentally. In remarkable contrast, the case of the airfoil having finite trailing-edge pressure gradients not only has a thinner displacement thickness at the trailing edge, but the slope discontinuities are largely eliminated.

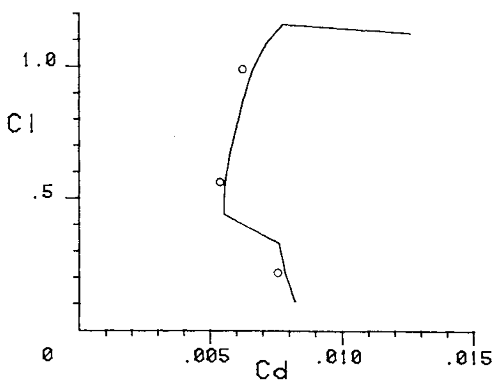

Fig. 6 Viscous analysis of Wortmann FX 67-K-150 using Eppler and Somers code; symbols are GRUMFOIL calculations; $Re = 2 \times 10^6$.

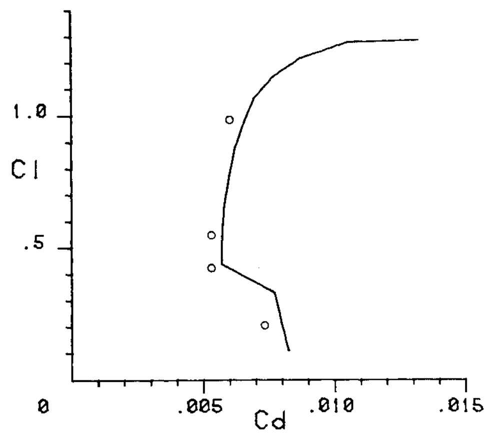

Fig. 7 Viscous analysis of airfoil based on Wortmann FX 67-K-150 using Eppler and Somers code; symbols are GRUMFOIL calculations; $Re = 2 \times 10^6$.

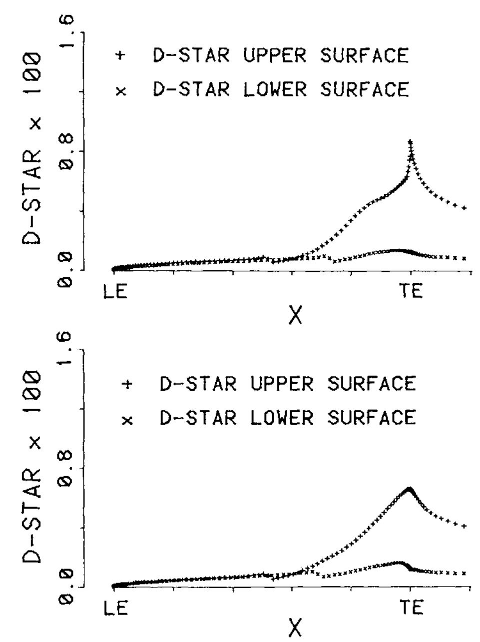

Fig. 8 GRUMFOIL calculation of displacement thicknesses for Wortmann FX 67-K-150 (top) and section based on that airfoil but having finite trailing-edge pressure gradients (bottom).

The ramifications of the results obtained using the GRUMFOIL code are significant. In addition to any performance benefits arising from smooth flow off the airfoil and into the wake, the application of the condition for finite trailing-edge pressure gradients has produced a class of airfoils for which strong viscous-inviscid interactions beyond those due to the displacement thickness are largely suppressed. That is, conventional boundary-layer theory remains valid in the region of the trailing edge for such airfoils and is sufficient for the prediction of their aerodynamic characteristics. Furthermore, as the influences due to viscosity are minimized, the results calculated using potential flow design methods should be more reliable than those generally obtained.

Concluding Remarks

The potential flow solution for any airfoil having nonzero trailing-edge loading is characterized by the presence of unbounded pressure gradients at the trailing edge. In the real fluid, a consequence of these singularities is the rapid growth of the boundary-layer displacement thickness in the vicinity of the trailing edge and its discontinuous slope at the trailing edge. Considering the near-critical nature of many of the velocity distributions prescribed for maximum lift or minimum drag, the encounter of such a disturbance could be sufficient to cause upstream separation problems. Thus, the realization of separation free flow should benefit by the removal of this disturbance, allowing the fluid on the airfoil to flow into the wake as smoothly as possible. Of additional concern in this regard is the presence of strong adverse pressure gradients in the vicinity of the trailing edge, as seen in many maximum-performance design efforts, which may also result in upstream separation problems. Thus, to help ensure that the high-performance levels promised by potential flow methods are achieved in practice, a procedure has been developed to design airfoils for which the trailing-edge pressure gradients are finite and the velocity distributions are free of strong adverse pressure gradients near the trailing edge.

In the formulation of conventional boundary-layer theory, normal pressure gradients through the boundary layer are ignored and, in application, only the influence of the displacement thickness on the inviscid results is considered. The presence of the unbounded pressure gradients that generally occur at the trailing edge in the inviscid solution, however, cause conventional boundary-layer theory to be invalid in the vicinity of trailing edge. In fact, it is found that the potential flow solution singularities give rise to additional viscous-inviscid interactions, each having an effect as important as that of the displacement thickness. By allowing for the influence caused by the normal pressure gradients in the trailing-edge region, wake thickness, and wake curvature, the GRUMFOIL code incorporates a self-consistent boundary-layer theory able to account for the strong viscous interactions due to the singularities in the inviscid flow solution. Although the formulation is distinctly different, the removal of the trailing-edge singularities can be considered an alternative approach to the same problem. In this light, airfoils having finite trailing-edge pressure gradients represent a class of airfoils for which the strong viscous-inviscid interactions in the trailing-edge region have been minimized. Consequently, conventional boundary-layer theory should be sufficient for the viscous analysis of the flow over such airfoils. Furthermore, because the corrections necessary to the inviscid solution due to viscous effects are minimal, potential flow design methods are likely to yield more reliable results than are otherwise obtained.

Considering the nature of the flow behavior in the region of the trailing edge, airfoils designed to have finite trailing-edge pressure gradients may be ideally suited to aid in the development and calibration of improved aerodynamic prediction methods for airfoils. For example, in the theoretical formulation used in the GRUMFOIL code, the trailing-edge region is locally modeled as unseparated flow over a flat plate at angle of attack. Thus, the class of airfoils having bounded pressure gradients at the trailing edge are much more consistent with this model than is generally the case. Such airfoils should, therefore, provide useful development tools and calibration cases. In a similar application, because the rapid growth of the displacement thickness near the trailing edge often leads to numerical divergence problems, the development of potential flow/boundary-layer iterative techniques should benefit from the well-behaved growth in the displacement thickness that occurs on airfoils having finite trailing-edge pressure gradients.

Finally, if imposing the requirement for finite trailing-edge pressure gradients does indeed minimize the viscous interactions and allow potential flow predictions to be more fully realized, then this situation clearly suggests that an improvement in airfoil performance is possible. In the example considered, the redesign effort was directed at matching the characteristics of a previously defined velocity distribution. Consequently, it remains to explore the potential of exploiting the use of finite trailing-edge pressure gradients to enhance airfoil aerodynamics. Encouragement that gains might be made is provided by the results of the GRUMFOIL analysis, which indicate, for example, that the employment of the finite trailing-edge pressure gradient condition yields an airfoil having a thinner displacement thickness and wake than occurs otherwise. If such characteristics can indeed be used to benefit performance, as the GRUMFOIL results indicate that reasonable off-design capability is present, airfoils having finite trailing-edge pressure gradients become candidates for practical application.

Acknowledgment

This study was funded by the NASA Langley Research Center under Grant NAG-1-76.

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